Probabilistic Correspondence Matching using Random Walk with Restart

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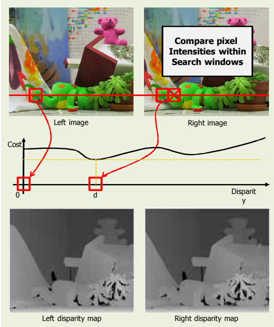
1. Introduction

Stereo Matching : Local Method

- Local Method

Compute correlation between points within a matching window

Multi-window methods	Choosing the best window among the predefined ones	Using local information → Low complexity Window shape is refined → Low performance
Adaptive weight method	Choosing the most similar window based on the color and spatial distance	Adaptive weight window → High performance Window size is large → High complexity
State-of-art methods	Reducing complexity with preserving the performance	High performance Low complexity



Local correspondence matching process

- Significance of the proposed method

Only an adjacent neighborhood (≈ 3x3 window) is needed
→ **Low complexity**
Meaningful steady-state distribution of each matching problems
→ **High performance**

2. Motivation

Re-formulation of local methods [1]

Local methods are mainly composed of three steps, and each step can be re-formulated as probability problem.

Conventional correspondence matching formulation

Matching cost computation: $e_0(\mathbf{x},\mathbf{d}) = \min(\|I_R(\mathbf{x}) - I_T(\mathbf{x},\mathbf{d})\|_1, \sigma)$

Cost aggregation: $e_{n+1}(\mathbf{x},\mathbf{d}) = \dfrac{\sum_{\mathbf{y}\in\mathcal{N}} w(\mathbf{x},\mathbf{y}) e_n(\mathbf{y},\mathbf{d})}{\sum_{\mathbf{y}\in\mathcal{N}} w(\mathbf{x},\mathbf{y})}$

Disparity computation: $\mathbf{d}(\mathbf{x}) = \underset{\mathbf{d}\in[\mathbf{d}_1,\ldots,\mathbf{d}_D]}{\arg\min}\, e_N(\mathbf{x},\mathbf{d})$

Probabilistic correspondence matching

Matching probability computation: $p_0(\mathbf{x},\mathbf{d}) = \max(\sigma - \|I_R(\mathbf{x}) - I_T(\mathbf{x},\mathbf{d})\|_1, 0)$

Cost aggregation: $p_{n+1}(\mathbf{x},\mathbf{d}) = \dfrac{\sum_{\mathbf{y}\in\mathcal{N}} w(\mathbf{x},\mathbf{y}) p_n(\mathbf{y},\mathbf{d})}{\sum_{\mathbf{y}\in\mathcal{N}} w(\mathbf{x},\mathbf{y})}$

Disparity computation: $\mathbf{d}(\mathbf{x}) = \underset{\mathbf{d}\in[\mathbf{d}_1,\ldots,\mathbf{d}_D]}{\arg\min}\, p_N(\mathbf{x},\mathbf{d})$

$\mathbf{x}=[x,y]^T, \mathbf{d}=[d,0]^T$ $w(\mathbf{x},\mathbf{y})$: *weight* I_R : *reference image* I_T : *target image*

-> **Probability optimization technique** can be used in correspondence matching problem.

3. Random Walk with Restart

Random Walk

The Random Walk (RW) theory has been widely used to optimize probabilistic problems. It has been known that the RW and the Laplace equation give the same solution, which means that **the steady-state of a given energy functional** can be captured by the RW.

steady state → **Trivial solution (Constant signal)**

Random Walk with Restart

The Random Walk with Restart (RWR) has become increasingly popular, since its **restarting term gives the meaningful information in a steady-state**, allowing it to consider the global relation at all scales. Note that the RWR becomes the RW as the restarting probability approaches to zero

steady state → **Non-trivial solution**

Relationship between Random Walk and Correspondence Matching

Inferring a probability with a small neighborhood is the same as a procedure of the RW, which means that the **adaptive weight method does not provide a meaningful steady-state solution similar to the RW**. Thus, in conventional methods, **the number of iteration should be specified** in advance and it significantly influences the performance of the algorithms

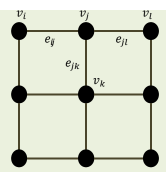
Disparity estimation results of the proposed method when the restarting probability is (from left to right) 3×10^{-1}, 3×10^{-3}, 3×10^{-5}, and 3×10^{-7}. As the restarting probability approaches to zero the results becomes constant, i.e. it has no meaningful solution.

4. Probabilistic Correspondence Matching

Graph model

Consider an **initial matching probability as an undirected graph** $G = (V, E)$ with nodes V and edges E. Each node $v_i \in V$ indicates a point at $x_i \in \{x_1, \ldots, x_M\}$ in an initial matching probability where M is the size of reference image. The adjacent nodes v_i and v_j are connected to an edge $e_{ij} \in E$. The graph assigns a weight to each edge as follows :

Edge weight: $w_{ij} = \exp\left(-\dfrac{\|I_R(\mathbf{x}_i) - I_R(\mathbf{x}_j)\|_2^2}{\gamma_c}\right)$



Graph model for an initial matching probability

4. Probabilistic Correspondence Matching

Probability inference

A random walker, with an initial position x_j, iteratively transits to its neighboring points according to the edge weight until it reaches to the reference position x_i. Also, the random walker goes back to x_j with the restarting probability at each iteration.

- Probability inference via RWR: Steady-state

- $\mathbf{P}_n^k = [p_n(\mathbf{x}_i, \mathbf{d}_k)]_{M\times1}$: Initial matching probability
- $\mathbf{W} = [w_{ij}]_{M\times M}$: Adjacency matrix
- $\mathbf{D} = diag(D_1,\ldots,D_M)$, and $D_i = \sum_{j=1}^{M} w_{ij}$
- α: Restarting probability

Formulation of iterative manner

$$\mathbf{P}_{n+1}^k = (1-\alpha)\mathbf{D}^{-1}\mathbf{W}\mathbf{P}_n^k + \alpha\mathbf{P}_0^k = (1-\alpha)\overline{\mathbf{W}}\mathbf{P}_n^k + \alpha\mathbf{P}_0^k$$

Steady state → Energy transition w.r.t. time approaches to 0

Formulation in steady-state

$$\mathbf{P}_s^k = (1-\alpha)\mathbf{D}^{-1}\mathbf{W}\mathbf{P}_s^k + \alpha\mathbf{P}_0^k = \alpha(I-(1-\alpha)\overline{\mathbf{W}})^{-1}\mathbf{P}_0^k = \mathbf{R}\mathbf{P}_0^k$$

: Probability transition term
: Initial probability restarting term
: All paths are considered when $\mathbf{P}_s^k$ is computed

- Probability inference via RWR: Consideration of R

- R : Interpreted as affinity scores between points in an initial state P_0^k

$$\mathbf{R} = \alpha(I-(1-\alpha)\overline{\mathbf{W}})^{-1} = \alpha\sum_{n=0}^{\infty}(1-\alpha)^n\overline{\mathbf{W}}^n$$ **: Infinite geometric series**

→ The iteration n denotes a scale of the transition, e.g., the random walker transits farther as n becomes larger.
→ Accordingly, the RWR optimizes an initial matching probability by considering all paths between two points at all scales.

Disparity computation

With a steady-state probability, a disparity can be simply selected by winner-takes-all strategy

$$\mathbf{d}(\mathbf{x}_i) = \underset{\mathbf{d}_k}{\arg\max}\, p_s(\mathbf{x}_i, \mathbf{d}_k)$$

Advantages

1) A non-trivial steady-state solution is guaranteed by constraining a steady-state probability to an initial matching probability to some extent, which means that it is not needed to specify the number of iteration.

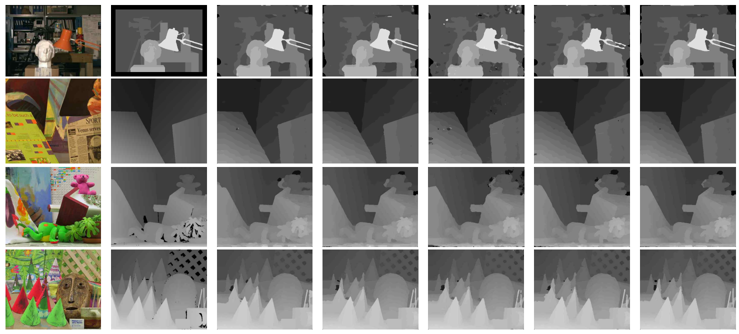
2) The global relationship between points or the steady-state solution can be captured by using an adjacent neighborhood only, which lowers the complexity of algorithm while maintaining the performance.

Accordingly, the proposed method gives high quality matching performance in a semi-global manner with low complexity

5. Experimental Results

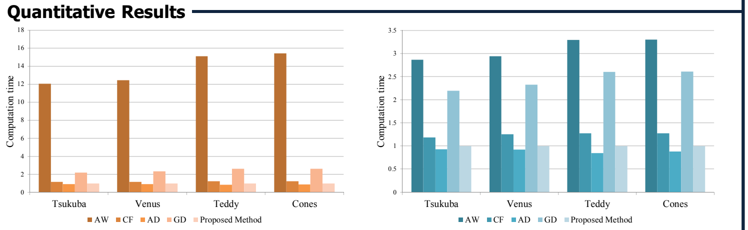
Qualitative Results

An inferred matching probability in 'Tsukuba' image when the candidate of disparity is 0. (From left to right) Original image, initial matching cost, Adaptive weight (AW) [2], Cost filter (CF) [3], Anisotropic diffusion (AD) [4], Geodesic diffusion (GD) [5], and (e) The proposed method. The low intensity indicates high probability of being matched, and vice versa.



Results for (from top to bottom) 'Tsukuba', 'Venus', 'Teddy', and 'Cones'. (From left to right) Reference image, Ground truth, Adaptive weight (AW) [2], Cost filter (CF) [3], Anisotropic diffusion (AD) [4], Geodesic diffusion (GD) [5], and The proposed method.

Quantitative Results



The computation time of disparity estimation with (from left to right) an optimal window and the smallest window (≈ 3x3 window). Note that the computation time of the proposed method is normalized to 1.0.

OBJECTIVE EVALUATION

Algorithm	*Tsukuba*			*Venus*			*Teddy*			*Cones*		
	Nonocc	*all*	*disc*	*nonocc*	*all*	*disc*	*nonocc*	*all*	*disc*	*nonocc*	*all*	*disc*
AW [2]	2.43	2.77	11.6	0.24	0.46	2.45	7.90	13.2	17.8	3.36	8.60	8.29
AW [2]*	1.38	1.85	6.90	0.71	1.19	6.13	7.88	13.3	18.6	3.97	9.79	8.26
CF [3]	1.76	2.14	8.34	0.19	0.46	2.51	6.24	11.5	16.0	2.48	8.01	7.20
CF [3]*	1.51	1.85	7.61	0.20	0.39	2.42	6.16	11.8	16.0	2.71	8.24	7.66
AD [4]	2.93	3.85	11.6	1.09	1.78	11.9	8.78	14.2	19.9	3.19	8.83	9.13
GD [5]	2.39	2.96	11.5	0.25	0.45	3.31	7.28	12.4	17.7	3.12	8.65	8.98
GD [5]*	1.88	2.35	7.64	0.38	0.82	3.02	5.99	11.3	13.3	2.84	8.33	8.09
Proposed Method	1.60	1.97	6.44	0.20	0.38	2.51	6.15	11.5	15.8	2.60	7.92	7.48

The symbol '*' indicates the results at the Middlebury test bed : http://vision.middlebury.edu/stereo

References

[1] D. Min, J. Lu, and M. N. Do "A revisit to cost aggregation in stereo matching: How far can we reduce its computational redundancy?," in *Proc. International Conference on Computer Vision*, pages 1567-1574, 2011.
[2] K. Yoon and I. Kweon, "Adaptive support-weight approach for correspondence search," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 28(4):650-656, April 2006.
[3] C. Rhemann, A. Hosni, M. Bleyer, C. Rother, and M. Gelautz, "Fast cost-volume filtering for visual correspondence and beyond," in *Proc. IEEE Conference on Computer Vision and Pattern Recognition*, pages 3017-3024, 2011.
[4] D. Min and K. Sohn, "Cost aggregation and occlusion handling with WLS in stereo matching," *IEEE Transactions on Image Processing*, 17(8):1431-1442, August 2008.
[5] L. De-Maeztu, A. Villanueva, and R. Cabeza, "Near Real-Time Stereo Matching Using Geodesic Diffusion," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 34(2):410-416, February 2012..